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MAGNETOSTRICTION OSCILLATORS.

AN APPLICATION OF MAGNETOSTRICTION TO THE CONTROL OF FREQUENCY OF AUDIO AND RADIO ELECTRIC OSCILLATIONS, TO THE PRODUCTION OF SOUND, AND TO THE MEASUREMENT OF THE ELASTIC CONSTANTS OF METALS.

By George W. Pierce.

With Three Plates.

MAGNETOSTRICTION OSCILLATORS

By George W. Pierce

Presented January 11, 1928. Received February 15, 1928.

Outline.—The present paper describes a newly-discovered method of using magnetostriction to produce and control electrical and mechanical frequencies of oscillations in a range of frequencies extending from a few hundred cycles per second to more than three hundred thousand cycles per second. The method involves the interaction of the mechanical vibrations of a magnetostrictive rod and the electric oscillations of an electric circuit in such a way that the electric currents in the circuit stimulate the rod to longitudinal vibration by magnetostriction and the vibrations of the rod react by magnetostriction on the electric circuit to maintain constancy of frequency.

The constancy of frequency obtained compares favorably with that obtained with the piezo-electric crystal oscillators. For ease of construction and operation the magnetostriction oscillator has a great advantage over the piezo-electric oscillator, particularly in that the construction and adjustment of the magnetostriction vibrators is so simple that large numbers of standards of frequencies all operable with the same electrical circuits may be had at small expense.

The magnetostriction oscillators supply a particularly pressing need in the range of frequencies below twenty-five thousand cycles per second in which range crystal-control is impractical on account of the expense of obtaining sufficiently large crystal vibrators. In the range between twenty-five thousand cycles per second and three hundred thousand cycles per second the magnetostriction oscillators and the crystal oscillators have a common field of usefulness. At frequencies greater that three hundred thousand cycles per second, the magnetostriction oscillators (although active up to two million cycles per second) are feeble with the present arrangement of apparatus. Their harmonics may, however, be employed up to frequencies of several millions per second.

This account contains also methods for calibration of the vibrators and their use in the calibration of wavemeters and frequency meters, data on the velocity of sound in various metallic alloys, data on the elastic constants of metals, including their temperature coefficients, description of methods of sound production, and a theoretical investigation of sound propagation in a viscous magnetostrictive medium.

Circuits and Mounting of Magnetostriction Oscillator for Controlling Oscillations in a Vacuum-Tube Circuit.—As shown in Fig. 1, two coils are employed, one, L_1, in the plate circuit and the other, L_2, in the grid circuit of a high-mu vacuum tube. The magnetostriction rod, R, which we may call the vibrator, is placed axially within the coils, and rests centrally on a support between the coils, or if

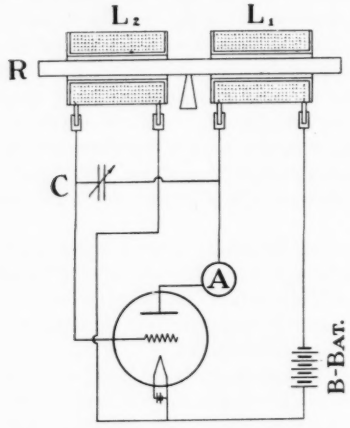


FIGURE 1. Oscillator Circuit.

desired is held in a clamp at the support. In the diagram the right-hand coil is in the plate circuit in series with a B-battery. The left-hand coil is in the grid circuit. A variable air condenser C is connected between the plate and grid, so as to be across both coils. Sometimes the condenser C is connected across only one of the coils.

By means of plugs and sockets the double-coil unit may be replaced by others of different inductances. With a given coil unit many different rods may be used by merely pulling out one and putting in another. The rod is free within the coils, which may have a clearance as large as the diameter of the rod itself. The rod is magnetized permanently or by the plate current, or by a permanent magnet placed near it.

A direct current milammeter at A serves to indicate the plate current and by its change with change of C to indicate the presence of oscillations.

Reversed Coil Note.—One fact is to be noted that for the best operation of the apparatus as a constant-frequency device, the coils are so wound and oriented that a steady current flowing from the filament to the plate and a steady current, if there were one, flowing from the filament to the grid produce magnetic fields in the same direction along the axis of the coils. This sense of the winding of the coils with respect to each other is opposite to that of the familiar electric oscillator circuits; but it should be noted that such an arrangement as that here shown, with certain values of circuit constants, will oscillate electrically without the rod or with the rod restrained.

Operation of the Magnetostrictive Rod in Rendering the System Oscillatory.—A system such as is shown in Fig. 1, may be entirely non-oscillatory when the rod is restrained from vibration by being held or when the condenser C has a value far removed from the value required to give the circuit a period near the period of vibration of the rod. In such a case when the rod is released and the condenser has, or is made to have, the proper value, the rod and circuit fall into oscillation with a frequency which is essentially the frequency of the rod, and the frequency remains practically unchanged even when the condenser is varied over a large range or removed altogether. With proper choice of the coils the condenser is unnecessary.

Numerical data as to the constancy of frequency will be given below.

The existence of the oscillations is evidenced by the sound emitted by the vibrating rod, if its frequency is within the audible range; or, whether audible or not, the vibration is evidenced by the change of direct-current indication of the plate milammeter. When the rod is allowed to vibrate the plate current changes to double or triple the value it has when the rod is restrained.

Operation of the Magnetostrictive Rod in Stabilizing the Frequency of an Electrically Oscillatory System.—If we consider the circuit of Fig. 1, and especially if we note that the electrical feed-back between grid coil and plate coil is the reverse of that usually employed in producing electrical oscillations, we may understand that with a given choice of condenser, coil-winding and coil-spacing, the system may or may not be electrically oscillatory when the magnetostrictive rod is restrained from vibration.

In the preceding paragraph note is made of the action with a non-oscillatory electrical system, which is the preferable mode of operation of the magnetostrictive system when employed with low fre-

quency vibrations of say five hundred to three thousand cycles per second.

At higher frequencies ranging from three thousand to three hundred thousand cycles per second, it is more convenient, and just as reliable, to allow the system to be electrically oscillatory even when the rod is restrained, and to employ the magnetostrictive rod merely to stabilize an already existing frequency of the electrical system when the latter is independently adjusted to a value near resonance with the period of mechanical vibration of the rod. This is done as follows: Any given rod, say one of frequency of twenty thousand, is placed in the coils. The plate milammeter is observed as the condenser C is varied. At a certain value of C, the plate current suddenly jumps to a large value (from one and one-half to three times its previous value). This indicates the incidence of rod-vibrations. The condenser may now be adjusted through a considerable range with no material change of the electrical or mechanical frequency of the system. It may be left oscillatory for days without changing frequency although the electrical characteristics may undergo large changes in that time.

General Note on Materials of the Magnetostrictive Rods.—This research, which has now extended over some years, has concerned itself largely with the study of materials for the vibrators. The qualities required are large magnetostrictive effects, and constancy of mechanical frequency in spite of changes of temperature and of intensity of magnetization, coupled with constancy of frequency in spite of changes of condenser settings, vacuum tube characteristics, plate currents and filament currents.

For these purposes pure iron and irons with various carbon contents are relatively useless, as having too small a magnetostrictive effect.

Pure nickel on the other hand is a good vibrator with, however, some lack of stabilizing power in that detuning slightly affects the frequency.

Alloys of nickel and iron in certain proportions are good vibrators, especially those having about 36% nickel and 64% iron (that is of about the constitution of *invar* and *stoic* metal). This alloy has, however, a large temperature coefficient of frequency.

Alloys of chromium, nickel and iron are good vibrators, and for many purposed commercial *Nichrome* is one of the best materials easily available.

An alloy of nickel and copper known as Monel metal, containing

68% Ni, 28% Cu, and small percentages of Fe, Si, Mn, and C, is a very powerful oscillator. It usually has too small a residual magnetism to oscillate without an auxiliary polarizing means.

Alloys of cobalt and iron are strong vibrators.

Tubes of nickel make good oscillators for qualitative work and demonstration purposes and for sources of sound but are somewhat lacking in constancy as frequency-stabilizers. Such tubes of nickel wholly or partly filled with lead or type metal permit the easy construction of low-frequency vibrators because the velocity of sound in lead is small and gives a low frequency of longitudinal vibration without excessive lengths of the rods.

By using a tube of material that has a negative temperature coefficient of frequency (such as nickel), in combination with a tight-fitting internal core of metal that has a positive temperature coefficient of frequency, (such as stoic metal), I have made composite vibrators of frequency practically independent of temperature. Other composite vibrators are described below.

All of the vibrators are made more powerful by annealing.

Mechanical Vibrations Longitudinal.—In most of the applications of the magnetostrictive vibrator to the production and stabilization of frequency I have employed the longitudinal vibrations of the rod either in its fundamental mode or some harmonic mode of longitudinal vibration. In the designation of constants in this paper, unless otherwise stated, reference is to the fundamental mode of longitudinal vibration. It is usually not worth while to use harmonic modes of vibration of the rods since new rods of any desired frequency can be made and calibrated with such ease that the harmonic mechanical vibrations are interesting only in studies of the mechanics of the rod itself. Investigations of this character are now under way.

By sending a current lengthwise through the rod so as to give a circumferential magnetic field and at the same time applying a current to the solenoid it is theoretically possible to produce torsional oscillations in the rod and these may be used to stabilize frequencies. I have found evidence of these torsional vibrations, but because of the complication of apparatus involved, these torsional oscillations are at present left out of the account.

Vibrators Magnetically Polarized.—The rods are polarized by a steady state of magnetization upon which the oscillatory magnetizing forces are superposed, so that the resulting state is an increase of magnetization when the oscillatory current is in one direction and a decrease of magnetization when the oscillatory current is in the

other direction. These two states are accompanied by an increase (or decrease) of length and a decrease (or increase) of length respectively. If the rod were not thus polarized the change of length would be the same for each half cycle of the current and would result in a mechanical vibration of double periodicity.

The polarization of the rods is usually attained by permanently magnetizing them in advance by a solenoid traversed by a strong direct current. With most substances the remanent magnetism is sufficiently strong to keep the rods sensitive as oscillators through years of normal use as standards of frequency in spite of any demagnetizing effects of the oscillations. The plate current of the tubes also assists in the polarization provided the rods, if permanently magnetized, are always inserted with their poles in the proper direction, as may be indicated on the coil mounting.

In cases of metals, like Monel metal, which have very small residual magnetism, I find that a small horse-shoe permanent magnetic placed in the vicinity of the rod is sufficient to give the required polarization, even when the small horse-shoe permanent magnet is 15 centimeters from the rod. With large oscillatory currents the polarization should be correspondingly large so that reversals of magnetization do not occur.

First Step in Explaining Action of Magnetostrictive Rod.—Along with the description of the apparatus occasional paragraphs are introduced explanatory of the theory of operation, with a final mathematical treatment of the system. Magnetostriction is the distortion of a body (in this case the lengthening or shortening of the rod) when magnetized. A rod of nickel when magnetized shortens by about one one-millionth of its length for a magnetizing field of one gauss. This is the observed fact when the magnetization is produced by a steady magnetizing force. This shortening is thus very small, for the reason that the shortening must take place against the enormous elastic force of the body. On the other hand, when the specimen is magnetized by a force that increases and decreases in an oscillatory manner at a period resonant with the period of the body the shortening and lengthening may be more than one hundred times as great as that obtained with the same magnitude of constant current applied to the specimen. In the resonant oscillatory case the contraction and expansion are no longer opposed by the elastic force of the body but only by its viscosity. We may call the decrease or increase of length under magnetization the *direct effect*.

There is also an *inverse effect*. When a magnetized bar is stretched mechanically its state of magnetization is changed.

We may now apply these facts to the system of Fig. 1. Any small fortuitous change of current through the plate coil L_1 changes the magnetization of the rod and causes it to be deformed (lengthened or shortened). This deformation is propagated along the rod to its left-hand end and exists temporarily as a deformation within the coil L_2. The deformation changes the state of magnetization and consequently induces an electromotive force in L_2. This acts on the grid and produces an amplified current change in the plate circuit and in L_1. The oscillating currents in the system thus build up to a large amplitude with a frequency determined by the frequency of longitudinal mechanical vibration of the rod.

Sample Exhibit of Rods.—Plate I is from a photograph of several rod vibrators. *G* is a set of *Stoic* metal vibrators with a range of frequencies extending from ten thousand to thirty thousand cycles per second at intervals of one thousand cycles per second. These are described in Table 1. *H* is a set of rods of *Nichrome* for each thousand cycles extending in range from twenty-six thousand to fifty-seven thousand cycles (see Table II). *F* is a rod of Nichrome with a frequency of 117,705 cycles per second. *D* shows a nickel tube filled with type metal. On account of the low velocity of sound in the type metal the specimen has a frequency of two thousand cycles per second, while a solid nickel rod or a nickel tube of the same length would have a frequency of nearly four thousand. *E* is a part of a similar tube, which is 94.35 cm. long, and is filled with lead. It has a frequency of one thousand cycles per second, which can be adjusted over a range of some seven cycles by clamp weights that may be moved nearer together or farther apart to get a higher or lower frequency. By using these movable weights this rod or a harmonic of it can be brought to beat zero with other standards. *C* is a solid oscillator of stainless steel (about 11 per cent chromium). It has a frequency of 23,640 cycles per second, and is here shown to call attention to the fact that rods of large diameters do not fail to vibrate even at high frequencies. In fact the specimen *A* is a thin disc between corks in a glass tube and while 1.5 cm. in diameter is only about 2 millimeters long and has a period of about a million and a half cycles per second. It is, however, a very feeble oscillator and evidences its oscillations only by transient clicks heard in a telephone in the plate circuit as the tuning condenser passes through resonance.

At *B* is shown a Nichrome oscillator of frequency 102,000 cycles per second in a sealed glass tube in vacuo, in which it is loose. In

the picture gravitation has pulled the oscillator to the lower end of the inclined glass tube. In operation the glass tube is placed axially in a horizontal position in the coils of the oscillator, and the rod is then centrally supported within the glass tube and within the coils. The mounting in a sealed glass tube has the advantage of permanence and avoids heating of the rod by the body temperature when it is handled. The absence of air or gas in the tube prevents the production of stationary sound waves in the tube, which might slightly affect the period of the rod, but this effect is very small.

A great variety of rod sizes are in use in various researches now under way at this laboratory.

Lengths and Frequencies of Rods.—Table I contains data obtained with a set of *Stoic* metal rods. These are the rods shown at *G* in the photograph. The frequency values given in the second column are accurate to 1/100 of 1%. The lengths are not accurately known because the ends of the rods are not accurately true. The errors in the measurements of these lengths are of the order of 0.1 mm. which amounts to more than 1/10 of 1% for the shorter rods.

It is seen that the length of the rods times their frequencies give a constant to the degree of precision to which the lengths are measured. This constant, when doubled, gives the velocity of sound in the metal, which is

$$v_{\text{stoic}} = 4160 \pm 2 \text{ m/sec at } 20^\circ \text{ C.}$$

All of these rods have roughly the same diameter, and the fact that there is no progressive change in the values of the last column indicates that there is no appreciable end-correction for the velocity determination.

The importance of this result for the present purpose is that a whole series of vibrators can be precomputed as to length with sufficient accuracy to make their final adjustment to required frequency very simple. A rod can be predetermined and cut in a lathe to 0.01 cm. so that the final frequency adjustment to a standard value requires very little grinding. The end is ground off to raise the frequency. If too much is ground off at the end, it can be corrected by grinding away a little from the girth near the center of the rod.

Table II contains a similar set of observations of frequency for different lengths of Nichrome. Velocities for a large number of alloys are given below.

TABLE I.

Length and Frequency of Stoic Metal Rods at 20° C.
Diameter 0.79 cm.

Specimen No.	Frequency Cycles/sec.	Length Cm.	Length in M. × Frequency
10	10001	20.815	2081.7
11	11005	18.91	2081.0
12	11985	17.36	2080.6
13	12990	16.01	2079.7
14	14000	14.85	2079.4
15	14981	13.87	2077.8
16	16009	13.00	2081.2
17	17006	12.24	2081.5
18	18015	11.55	2080.7
19	19013	10.94	2080.0
20	20003	10.40	2080.3
21	21007	9.90	2079.7
22	22008	9.44	2077.5
23	22970	9.035	2075.2
24	24009	8.65	2076.8
25	24992	8.33	2081.8
26	25979	8.01	2080.9
27	26981	7.70	2077.5
28	27966	7.44	2080.5
29	29000	7.17	2079.2
30	29981	6.93	2077.7
		Average	2079.6 ± 2

$v_{stoic} = 2 \times \text{Length} \times \text{Frequency} = 4160 \pm 2$ meters/sec. at 20° C.

Experimental Data Illustrating the Manner in which the Magnetostrictive Rod Functions in Controlling Frequency.—Illustrative of the action of the rod as a frequency-controlling element the experimental curves of Fig. 2 are given. These are from observations taken with a rod (No. 17) of Nichrome in the coils and circuit of

TABLE II.

Length and Frequency of Nichrome Rods at 23° C. Diameter .96 Cm.

Specimen No.	Frequency Cycles/sec.	Length Cm.	Length in M. × Frequency
26	25999	9.58	2490.7
27	27000	9.21	2484.0
28	28000	8.87	2486.4
29	29005	8.56	2482.8
30	29992	8.27	2480.3
31	31005	8.02	2486.6
32	32011	7.77	2487.2
33	33006	7.55	2491.9
34	34011	7.32	2489.6
35	35002	7.13	2495.6
36	36002	6.88	(2476.9)*
37	37004	6.73	2490.3
38	38004	6.55	2489.2
39	39005	6.38	2488.5
40	40004	6.23	2492.2
41	41007	6.06	2485.0
42	42000	5.93	2490.6
43	43005	5.79	2490.0
44	44009	5.66	2490.9
45	45008	5.53	2488.9
46	46004	5.42	2493.4
47	47009	5.30	2491.4
48	48011	5.20	2496.5
49	49008	5.09	2494.5
50	49996	4.99	2494.8
51	51040	4.87	2485.6
52	52000	4.78	2485.6
53	53062	4.69	2488.6
54	54027	4.61	2490.6
55	55014	4.53	2492.1
56	56042	4.45	2493.9
57	57015	4.37	2491.5
		Average	2490.3

$v_{nichrome}$ = 4981 m/sec. at 23° C.

* Inconsistent on account of grinding at girth for adjustment.

Fig. 1. In the lower part of the Fig. 2 electric wave length λ, read on a precision wavemeter, are plotted against condenser divisions of the condenser C of Fig. 1. Readings were taken first with the rod restrained by holding it with the hand. A plot of λ versus C for the rod thus restrained is the curve ABCDE of Fig. 2.

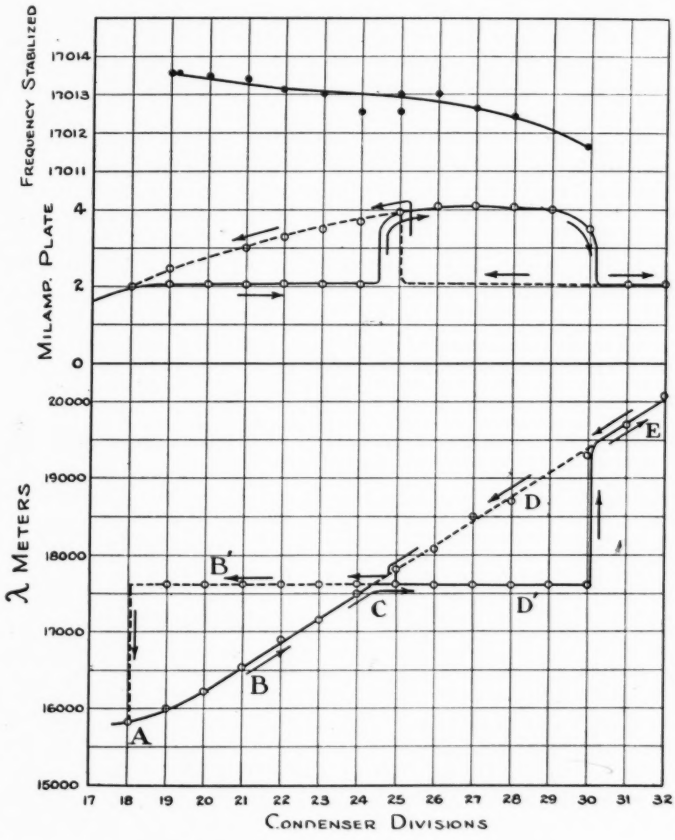


FIGURE 2. Illustrating Stabilization of Frequency.

Next readings were taken with the rod free, starting with small values of the condenser C. These readings plotted give the curve ABCD'E, with a constant λ along the element D' of the path.

As a third operation, with the rod free the condenser was started at large values of condenser and gave the curve EDB'A, with constant λ along the element B' of the path.

As a fourth operation it is noted that when we have once got the rod vibrating by either of the above operations we can pass back and forth along the whole of B' and D' (i. e. from 18 divisions to 30 divisions of the condenser) and the wavelength remains essentially constant.

The wave-meter measurements are not sensitive enough to detect any change of frequency along this path B'D'. To measure this quantity resort was had to beats between the oscillating system with rod No. 17 under investigation and a second oscillator with magnetostriction-rod control. Since the total change of No. 17 was of the order of one cycle per second in seventeen thousand, the second oscillator, known as No. 117.7, was chosen to have a frequency about seven times that of No. 17, and beats were observed between the seventh harmonic of No. 17 and the fundamental of No. 117.7. This means that one cycle per second change in No. 17 gives seven cycles per second change in the beat note, which can be measured to about one cycle per second.

With this device the actual frequencies of the system stabilized by rod No. 17 against condenser settings are plotted as the top curve of Fig. 2. This curve shows that a change of the condenser from 19 to 30 divisions changes the frequency of the system by only 1.5 parts in seventeen thousand. This effect is discussed further in the next section.

In this experiment I have changed the condenser through a wide range of values, so as to make the change of frequency as much as possible, but when used as a constant frequency apparatus the condenser is brought always to the value which makes the plate current a maximum so that repeated settings of the apparatus may be made with a much higher degree of precision that the 1.5 cycles in seventeen thousand cycles per second.

Let us look next at the middle curves of this Fig. 2, which exhibits the plot of plate current against condenser settings, when the rod is free to vibrate. The arrows indicate the sense of a change given to the condenser of the oscillator circuit. When the condenser, on being increased, reaches 24.5 divisions the rod begins to vibrate and

the plate direct current jumps from two to four milamperes, and remains about the same for further increases of the condenser up to 30.1 divisions when the rod ceases to vibrate. Now a reduction of the condenser leaves the plate current two milamperes till $C = 25$ divisions is reached when the plate current jumps again to four milamperes and then gradually decreases as the condenser is changed further down to eighteen divisions, when the rod is found to have again ceased to vibrate. In fact with this particular set of apparatus constants the system ceases also to vibrate electrically at C equal to or less than eighteen divisions. This accounts for the smooth fall of plate current to its constant two milampere value. With more clearance in the coil between the rod and the coil or with other electrical or mechanical constants we may get an abrupt change of plate current at the left hand part of the curves as well as at the right hand part. With less clearance, or by putting a separate condenser about each of the coils of Fig. 1, the apparatus can be so constructed that all oscillations cease whenever the rod is restrained from vibrating.

The Apparent Reactance of the Coil Containing the Tuned Magnetostriction Rod a Function of the Frequency.—A crude view of the above result is that in the range of frequency-stabilization the system oscillates as a condenser-inductance circuit with an inductance that varies enormously with a small change of frequency. If we regard the wavelength-condenser curves of the lower part of Fig. 2 as having the equation

$$\lambda_d = 2\pi c \sqrt{L_d (C + C_o)} \text{ for the damped curve}$$

and

$$\lambda_f = 2\pi c \sqrt{L_f (C + C_o)} \text{ for the free curve}$$

we obtain

$$\left(\frac{\lambda_f}{\lambda_d}\right)^2 = \frac{L_f}{L_d}.$$

The values of this ratio are plotted against frequency in the curve of Fig. 3, from which it appears that the inductance of the coil and free rod, in the range of its stabilization frequencies, changes by about 50% for a frequency change of 1.5 in 17013; that is, for a frequency change of less than 1/100 of 1%.

Though the correct understanding of the device requires mathematical discussion, such as is given below, we may here regard the constancy of frequency, with large variation of condenser, as due to

the fact that an increase of C makes the frequency lower. This is accompanied by a shift of L along the curve of Fig. 3 to a smaller value so that the product of $L \times C$, and consequently the frequency, is almost the same as before the increase of C and, in fact, with this particular magnetostrictive material (Nichrome) the frequency changes only 1/100 of 1% for a change of 50% in the value of C. The potency of the rod in controlling the frequency is seen by noting

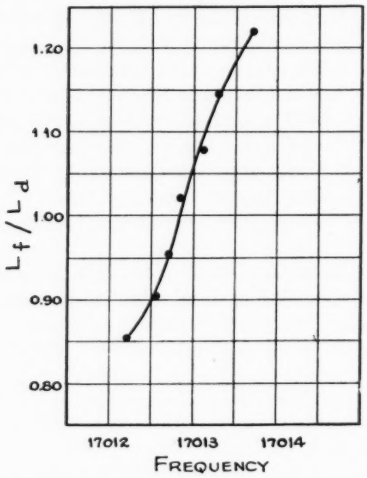


FIGURE 3. L_f = Inductance Free; L_f = Inductance Damped; plotted against Frequency.

that if we omit the rod or restrain it from vibrating this change of 50% in C causes 25% change in frequency instead of the 1/100 of 1% with the rod present and free.

Frequency Essentially Independent of Vacuum-Tube Voltages and Characteristics.—I have made numerous experiments that show that the very large change of plate voltage from 135 volts to 67 volts changes the frequency of oscillation of the magnetostriction controlled system by only about one in thirty thousand. Voltage change of the filament battery from a condition of practically zero emission to the point of destruction of the filament, or a change of the inductance of driving coils over the whole range of oscillatory

operativeness or a change of the tubes to tubes of widely different characteristics affects the frequency within about the same limits.

These various notes as to constancy apply to annealed Nichrome rods. Some of the other materials and particularly certain tubular vibrators are not quite so constant in these particulars, but are still very serviceable when less precision is required.

Temperature Coefficient of the Nichrome Vibrators.—The temperature coefficient of the frequency of the Nichrome vibrator is

$$\frac{\Delta f}{f \Delta \theta} = - .000107/\text{deg. C.}$$

where f = frequency, Δf = increase of frequency for a change of temperature of $\Delta \theta$ degrees Centigrade. This equation means that the frequency decreases about 1/93 of 1% per degree Centigrade.

The temperature coefficient of other materials is given in a later section.

Comparison of Constancy with that of the Piezo-Electric Crystal Vibrators.—For the purpose of comparison of the magnetostriction oscillators with the piezo-electric crystal method of frequency control to which I have contributed in previous researches I have made extreme tests on some of my best piezo-electric crystal frequency standards. One is a crystal, No. 28, mounted in vacuo in an accurately designed and carefully machined holder. When operated under constant conditions its absolute frequency has been repeatedly measured and shows variations below 1/500 of 1%. However, care must be taken to keep the circuits constant with this crystal oscillator as the data of Table III show.

TABLE III.

Piezo-Electric Crystal Oscillator

Conditions		Frequency
Tube	Plate Volts	
199	45	28072
Daven 20	67	28064
Daven 20	112	28061
Daven 20	135	28058
201A	90	28056
201A	112	28054
201A	135	28052

It is seen that extreme variations of tubes and plate voltage changes the frequency of this crystal oscillator by 7/100 of 1%.

This crystal also changes frequency to about the same extent with change of the inductance and distributed capacity of the plate coil. Table III compared with previous sections of this account shows that for equally violent change of the circuits, tubes and voltages with the piezo crystal oscillator and the magnetostriction oscillator the magnetostriction oscillator is much more constant as to frequency than is the piezo crystal oscillator.

It should be noted however that in the matter of temperature coefficient of frequency the particular Nichrome magnetostriction oscillator with 1/93 of 1% per degree Centigrade is more subject to temperature changes than the piezo-electric oscillators with 1/200 to 1/1000 of 1% per degree Centigrade.

Below I give, however, descriptions of other magnetostrictive materials with temperature coefficients of about the same order as that of the piezo-electric crystal oscillators. The temperature effect in no case is permanent and the frequency comes back to its calibrated value when the temperature returns to normal for the calibration. A knowledge of the temperature permits the application of a correction to the rated calibration.

Magnetostriction Oscillator with Universal One-Stage Amplification.—In order to increase the output power one or more stages of amplification may be employed. Fig. 4 shows a very simple and useful arrangement for a system to be operated at frequencies extending all over the audio and radio frequency ranges. The oscillator tube shown at the left has the connections of Fig. 1. The grid of the amplifier tube at the right is connected through a condenser C_1 to the plate of the oscillator tube, and is provided with a leak resistance. The plate circuit of the amplifier tube contains a "speaker filter" unit consisting of an audio choke which is in series with the B-Battery and which communicates with the output terminals through a condenser C_2 of two microfarads.

The constants given in the legend to Fig. 4 are so chosen that the amplifier operates with considerable efficiency at both radio and audio frequency. In the latter case a loud speaker connected to the output terminals gives volume enough to be heard throughout a large lecture room. This is the case also with radio oscillations upon which are superposed other radio oscillations to produce audio frequency beats.

A commercial apparatus of this character made for me by the

General Radio Company of Cambridge is shown in Plate II, and at the left in Plate III. The coil units, containing rods, on top of the boxes are plugged into jacks and can be removed and interchanged. The upper cover of the box opens.

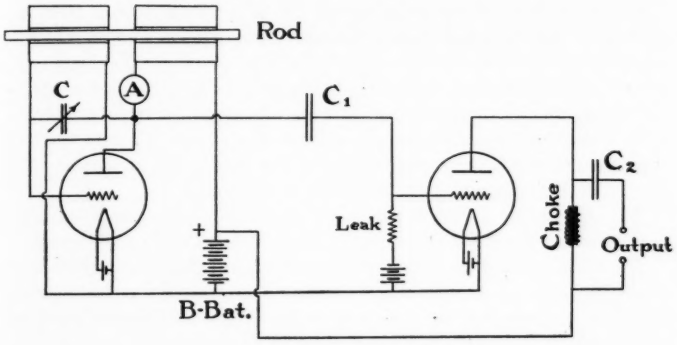


FIGURE 4. Magnetostriction Oscillator with One-stage Amplification. C_1 = 0.1 Microfarad, C_2 = 2 Microfarads, Leak = .05 Megohms, A = Milammeters.

Twin Magnetostriction Oscillator.—For calibration purposes we need two oscillators like that of Fig. 4, one of which is maintained constant by a standard magnetostriction rod, and the other of which is at a frequency to be compared with the rod. The two oscillators are then caused to interact and their beat frequency is determined or reduced to zero. After much experimenting I find that the best way to get the interaction is to connect their output terminals together through a small condenser and then to plug the telephones or other indicator in at one or the other of the output terminals. If the rod is left out of one of the coil units that half of the twin oscillator may be used as an ordinary variable electric oscillator.

I have found so much use for this arrangement that I have had both of these oscillators with common B-Batteries and common A-Battery terminals mounted in a single apparatus shown as the middle unit of Plate III.

The "speaker filters" of the two output circuits are connected with a switch so that if desired, either or both may be cut out and the B-Battery and plate of the amplifiers be connected directly to the

output terminals. This is useful in running a synchronous-motor clock, which requires a polarizing current, as the output apparatus. Such a clock made also by the General Radio Company from their own designs and kindly put at my disposal is shown in Plate II, and was used in timing some of the low-frequency rods (at about 1000).

Absolute Frequency Calibration by Magnetostriction Oscillators and the General Radio Synchronous-Motor Clock.—Use is made of the adjustable Magnetostriction Rod of about 1000 cycles per second, which is shown in Plate II mounted on top of the box containing the electric oscillator circuits and the one-stage universal amplifier. Used with the "Speaker filter" cut out this drives the motor clock synchronously with the oscillations of the rod. The amplifier tube is a U.X. 171, operating on 130 volts in the plate.

The synchronous clock has been provided with circuit-making contacts that close for a very short time each second of the clock.

For a standard of time with which to compare this motor clock Professor Stetson of the Harvard Student's Astronomical Laboratory has kindly permitted me to lead wires from his standard Riefler Clock which beats siderial seconds with an error not greater than two-tenths of a second per week. All relay contacting members are obviated in this circuit by the device of placing an inductance coil near the field coil of his relay, so that the local current from the time wheel of the clock through his relay field acts inductively on my circuit and supplies second impulses without the inconstancy of any contact other than that of the time wheel of the clock. These second impulses arriving at this laboratory are amplified by a one-stage amplifier, to the output of which is connected a loud-speaker through the contacting points of the motor driven clock. When the standard Riefler clock and the motor clock make contacts at the same time the ticks are heard in the loud speaker. By reading the motor clock for several successive coincidences accurate timing is obtained of the motor clock and of the magnetostriction driving rod.

For timing any other suitable magnetostriction rod, (say one of 102,000 vibrations per second) the latter is used to drive and control a second oscillator mounted on a second box like that of Plate II, or else the twin system is used with both of the rods. The 1,000 cycle rod is now adjusted by moving the weights out or in along the rod to points where zero beats is obtained between the 102nd harmonic of the 1,000 cycle rod and the fundamental of the 102,000 rod. When this adjustment is nearly perfect the remaining small adjustment is

made by changing the tuning condenser of the 1,000 cycle system. In this way I have succeeded in reducing the beat frequency to one cycle in five seconds, which means an error of this adjustment of the harmonic of the low frequency to the fundamental of the high frequency of two parts in a million. Such accuracy is ordinarily not required, so that more tolerance is usually given to this adjustment.

It is interesting to note that the beats may be observed on a D. C. milammeter as well as in a telephone. As a means of obtaining beats on the milammeter the circuits of Fig. 5 are recommended.

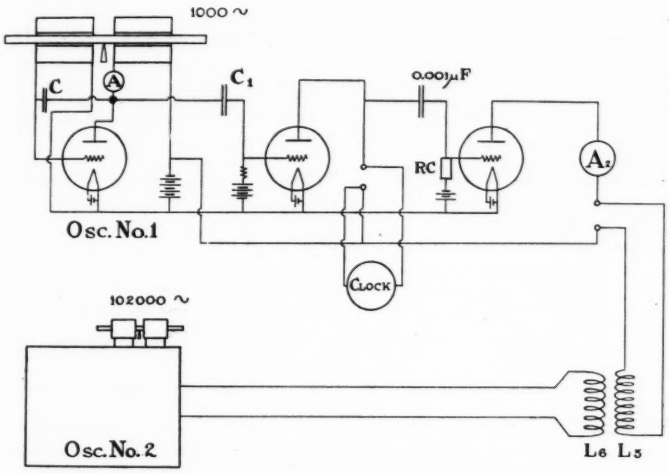


FIGURE 5. Circuits for Absolute Calibration of Frequency.

In Fig. 5 Oscillator No. 2 has the circuits of Fig. 4 with a coil L_6 connected to its output terminals. Oscillator No. 1 has the same circuits for the two left-hand tubes with, however, the motor-clock in place at the choke of the second tube. Paralleling the clock a radio-amplifying unit is employed for further amplifying the higher harmonics of Oscillator No. 1 and for suppressing its 1,000 cycle note and low pitch harmonics. The connection to the third tube is through a condenser of 0.001 microfarad, which has high impedance to these low notes. The customary grid-leak of this third tube is replaced by a radio choke *R C* of about 0.1 henry, which has a

small impedance for the low notes and a high impedance for the higher harmonics. The output circuit leads to L_5 inductively connected with L_6 so that the oscillations of the two systems are superposed in their output circuits, and their beats when nearly of zero frequency are observed on the D. C. milammeter A_2.

In this way timing of the motor-clock gives a direct standardization of the 102,000 cycle rod as well as of the 1,000 cycle rod.

In a similar way any of the rods may be calibrated directly in one step. I have used harmonics of the 1,000 cycle rod up to its 234th.

To calibrate at the same time directly any higher frequency a double step is required.

Examples of Absolute Calibration of Some of the Rods.—In calibrating Nichrome Rod No. 30, the Rod No. 1 (of about 1,000) was adjusted to give zero beats of its 30th harmonic with Rod No. 30. Coincidences of the Rod-Clock seconds with the Riefler clock siderial seconds occurred as follows:

Coincidences Read on Rod Clock			Time Between Coincidences in Rod Clock Seconds
hr.	min.	sec.	
3	14	31	396
3	21	17	395
3	27	42	399
3	34	21	

For computation we use

$$1 \text{ solar true second} = 1.00273 \text{ siderial seconds.}$$

$$1 \text{ rod second} = 1 + \frac{1}{397} = 1.00252 \text{ siderial seconds.}$$

$$\therefore \quad 1 \text{ solar second} = \frac{1.00273}{1.00252} = 1.00021 \text{ rod seconds.}$$

The motor clock operated by the rod has its hands so geared to its motor that its time is correct when the motor is driven by a current of 1,000 cycles per second, so we have for the frequencies of the rods:

$$f_{\text{no. 1}} = 1000.21, f_{\text{no. 30}} = 30{,}006.3 \text{ at } 20°.4 \text{ C.}$$

Two other independent determinations of this rod No. 30 give 30,007.2

and 30,006.9 at 20°.2 C. The average of the three values is 30,006.8, with an average error of ½ cycle. Each of these calibrations had a timing period of about 20 minutes; by prolonging the time any required greater accuracy can be obtained.

The following values, Table IV, taken at random from my notebook have been obtained in the absolute calibration of a number of standards, and illustrate what can be done with even very careless work provided the standard second is good and is applied sharply to the apparatus.

TABLE IV.

Sample Absolute Calibration of Standard

Standard No.	Rod No. 30 at 20°.3 C.	Rod No. 102 at 20°.6 C.	Rod No. 117.7	Crystal No. 5884
Cycles per Second	30006.3 30006.9 30007.2	102078 102085 102084 102084 102083 102084	117718 at 21°.9 C. 117679 at 24°.5 C. 117705 at 23°.0 C.	5884707 5884722
Average	30006.8	102083	117701 at 23°.C.	5884719

Referring to the values for Rod No. 117.7 the three values were taken at different temperatures and if reduced to a common temperature of, say 23°, these values become 117704, 117689, 117705 averaging 117701 with an average error of 2 cycles per second. With this correction for temperature the values for No. 117.7 have about the same consistency as the other values in the table, and the variations in the table amounting to about 1/500 of 1% are due to inaccuracy arising from the paucity of the coincidence observations.

The measurements of the frequency of Crystal No. 5884 of about 5884 kilocycles were obtained by using the rod of 1000 cycles adjusted to 997.28 cycles so that its 118th harmonic beats zero with rod No. 117.7 and at the same time beats were measured between the 50th harmonic of No. 117.7 and the fundamental of Crystal No. 5884. The latter number of beats per second was found to be 772, so we have $f_{5884} = 50\ (118 \times 997.28) + 772 = 5{,}884{,}722$. This double operation steps up the 1000 cycle frequency to its 5900th harmonic.

It is interesting to note that this Crystal No. 5884 can also be used at all of its harmonics up to its 30th harmonic to measure frequencies up 150 million cycles.

Mr. M. T. Dow, Dr. Frederick Drake and I are using such a calibration in the precision measurement of the velocity of electric wave.

By the methods here described the several rods of Plate I have been calibrated and intercompared. In the intercomparison the differences between fundamentals or harmonics of various oscillators may be conveniently measured by a bridge-type of audiofrequency meter such as is described in the next section.

Audiofrequency Meter.—I here call attention to a bridge-type of audiofrequency meter involving well-known circuits[1] that I have found useful for measuring audio-frequencies. A photographic view

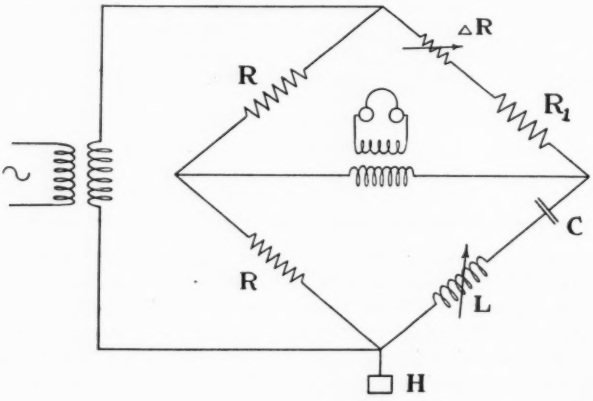


FIGURE 6. Circuits of Audiofrequency Meter.

of this apparatus is shown as the right-hand unit of Plate III. The circuits are shown in Fig. 6 and comprise a bridge of three resistance arms and one arm of a variable L in series with fixed C. The two resistances R R are equal. The resistance R_1 plus ΔR is equal to

[1] Although this instrument has been in use here for many years it was apparently previously described and used as a frequency meter by *Heydweiller and Hagemeister: Verh. d. D. Phys. Ges.* 18, p. 52, 1916. For other references see *Banneitz: Taschenbuch der drahtlosen Tel. und Tel.*

the resistance of the branch L C which is chiefly the resistance of L. Since L is of copper wire it has a temperature coefficient. This is compensated by the small adjustable resistance Δ R, which when necessary is independently adjusted when the bridge is to be balanced. The chief variable element is L. At any given frequency, f, within the range of the apparatus the balance is obtained when

$$2\pi f = \frac{1}{\sqrt{LC}}$$

Frequencies are read on a direct-reading scale calibrated in frequency attached to the dial of L. One important point is that for repeatable readings with an accuracy of 1 part in 1000 the variable effects of body-capacity must be eliminated. I have found it advisable to provide the inductance L with a metal knob so that when one takes hold of this knob to adjust L he connects his body to the point H in Fig. 6.

The inductance L at full scale is about 200 milhenries. The capacity C is a three-step mica condenser, having the three values 0.04, 0.2, and 1.0 microfarad, which gives the instrument a three-fold scale extending from 365 to 5000 cycles per second. The switch in the center of the photographic view of the instrument is for switching to different values of the condenser, and automatically switches the pointer to the proper direct-reading scale. The scale was drawn by hand on a bakelite disc from experimentally determined frequency readings and was then engraved on the disc.

In the instrument are included a 1 input transformer, for connecting directly into the input circuit of a vacuum-tube oscillator, and an output transformer connecting to head telephones or a loud-speaker.

Absolute Calibration of Audiofrequency Meter of Fig. 6 by Nichrome Rod No. 30 with f = 30,000.—An electric oscillator with frequencies variable to cover the range of the Audiometer is made to beat against various harmonics of the fixed 30,000-cycle frequency of the rod No. 30. Practically, in the present calibration, this was done by using the Twin Oscillator of Plate III (shown at the center) with the magnetostriction rod in its coil assembly plugged into one set of jacks on the instrument and with various coil-pairs without magnetostriction core plugged into the other set of jacks. One pair of the output terminals of the Twin Oscillator were now connected to the input posts of the audiofrequency meter to be calibrated. A telephone receiver was at the output posts of the audiofrequency meter.

The procedure is as follows: Adjust the variable electric oscillator to beat zero with some harmonic of the magnetostriction rod, then adjust the audiofrequency meter for silence in the telephone, and read the scale of the audiofrequency meter. Proceed *ad lib.* with other harmonics.

Table V gives the results. The value of the harmonic used is given in the column marked n. For example $n = 3/50$ means a beat zero between the 50th harmonic of the 30,000 cycles rod and the 3rd harmonic of the electric oscillator, to which the audiometer was set, giving $f = 1800$ cycles per second. This is tabulated in the second column.

The observations in this Table V were made hurriedly without any repetition of readings. Only the more striking harmonics (simple ratios) were used, and a large number of other values were passed over without being read, as there was no need of obtaining any more dense distribution of the calibration points. The accuracy of reading the scale of the instrument was only about 1 part in 1000 in the more open parts of the scale. Checks on the identity of the harmonics can be obtained by using other magnetostriction rods

Absolute Calibration of a Wavemeter by Nichrome Rod No. 30.— To show a further use of the magnetostriction standards this same rod of 30,000 cycles per second ($\lambda = 10{,}000$ meters) was used to obtain the values of Table VI in calibration of a wave-meter ranging from 233 to 2224 meters. The procedure was the same as in the preceding section, except that the electric oscillator coils were of small inductance so as to give high frequency. The telephone connected directly at the output of the Twin Oscillator, served to evidence audible beats, which were adjusted to a zero-beat-frequency.

The wavemeter was placed near the electric oscillator coil and had its coil shortcircuited, while setting the oscillator to beat zero. Then the shorting switch of the wavemeter was opened, and the condenser of the wavemeter was adjusted. As resonance was approached beats were again heard in the telephone. Changing the condenser of the wavemeter in one direction, we hear these beats rise in frequency, then drop to zero, then appear again at a high frequency and gradually change to zero. The drop of the beats to zero at the setting between the two high frequency beat values is the adjustment of the wavemeter to resonance.

In Table VI n indicates the harmonics employed. This is here tabulated as $1/n$, since we are concerned with wavelengths. The wavelength λ of the wavemeter is obtained by taking $1/n$ times the electric wavelength (10,000) of the 30,000 cycle rod.

TABLE V.

Calibration of Audiofrequency Meter vs. Nichrome Rod No. 30.

Scale of Instrument	f by No. 30	n	Scale of Instrument	f by No. 30	n
872	870	2/69	1760	1767	1/17
897	896	2/67	1800	1800	3/50
909	909	1/33	1818	1818	2/33
922	923	2/65	1871	1875	1/16
937	938	1/32	1909	1915	3/47
952	952	2/63	1935	1935	2/31
956	957	3/94	2000	2000	1/15
968	968	1/31	2068	2069	2/29
978	978	3/92	2140	2143	1/14
982	984	2/61	2223	2222	2/27
1090	1091	2/55	2305	2308	1/13
1130	1132	2/52	2395	2400	2/25
1181	1176	2/51	2500	2500	1/12
1228	1224	2/49	2605	2609	2/23
1252	1250	1/24	2722	2727	1/11
1278	1277	2/47	2999	3000	1/10
1330	1333	2/45	3165	3158	2/19
1390	1395	2/43	3330	3333	1/9
1462	1463	2/41	3535	3529	2/17
1498	1500	1/20	3740	3750	1/8
1543	1538	2/39	4000	4000	2/15
1581	1579	1/19	4285	4285	1/7
1603	1607	1/23	4478	4500	3/20
1620	1622	2/37	4595	4615	2/13
1660	1666	1/18	4710	4737	3/19
1692	1696	3/53	5010	5000	1/6
1710	1714	2/35			

In this section and the preceding section it is seen that the 30,000-cycle rod is used in frequency calibrations extending from 870 cycles to 1,200,000 cycles. The possible range is vastly greater than this.

TABLE VI.

Calibration of Wavemeter LC_1 vs. Nichrome Rod No. 30. Readings are in Meters Wavelength.

Scale of Instrument	λ by No. 30	1/n	Scale of Instrument	λ by No. 30	1/n
2224	2222	2/9	866	869	2/23
2175	2174	5/23	832	825	1/12
2144	2143	3/4	799	800	2/25
2106	2105	4/19	768	769	1/13
2085	2084	5/24	712	714	1/14
2070	2068	6/29	662	666	1/15
2000	2000	1/5	621	625	1/16
1940	1944	7/36	574	578	1/17
1924	1923	5/26	553	555	1/18
1892	1892	7/37	523	526	1/19
1870	1875	3/16	514	513	2/39
1848	1852	5/27	502	500	1/20
1815	1818	2/11	475	476	1/21
1790	1786	5/28	454	455	1/22
1712	1714	6/35	435	435	1/23
1668	1667	1/6	416	417	1/24
1573	1579	3/19	401	400	1/25
1535	1538	2/13	383	385	1/26
1510	1515	5/33	368	370	1/27
1500	1500	3/20	356	357	1/28
1480	1481	4/27	344	345	1/29
1462	1463	6/41	333	333	1/30
1426	1429	1/7	322	323	1/31
1388	1389	5/36	312	313	1/32
1378	1379	4/29	303	303	1/33
1362	1364	3/22	295	294	1/34
1350	1352	5/37	285	286	1/35
1332	1333	2/15	276	278	1/36
1303	1304	3/23	270	270	1/37
1251	1250	1/8	263	263	1/38
1201	1200	3/25	257	256	1/39
1178	1176	2/17	251	250	1/40
1111	1111	1/9	245	243	1/41
1050	1052	2/19	238	238	1/42
999	1000	1/10	233	233	1/43
952	952	2/21			

TABLE VII.

Experimental Results for Alloys.

v = velocity of sound, g = temperature coefficient of frequency, ρ = density, E = Young's Modulus, a = coefficient of expansion, h = temperature coefficient of velocity, b = temperature coefficient of elasticity.

Material	v m/sec	g $\times 10^6$	ρ g/cc	$E \times 10^{11}$ Dyne / sq .cm.	a $\times 10^6$	h $\times 10^6$	b $\times 10^6$
Iron	5074	−171	7.688	19.79	12	−159	−354
Nickel	4937	−132	8.803	21.46	12	−120	−276
Stoic	4161	+224	8.02	13.89	1.5	+226	+446
Nichrome	4981	−107	8.269	20.52	12	− 97	−226
Monel	4549	−151	8.854	18.32	14	−137	−316
Stainless Steel	5430	−136	7.720	22.76	10	−135	−282
Stainless-Iron	5133	−130	7.743	20.40	10	−130	−270
Nickel-Iron % Ni							
0	5074	−171	7.688	19.79	13.3	−158	−355
10	4919	−164	7.812	18.90	9	−155	−337
20	4582	−159	7.818	16.41	8	−151	−326
30	4527	+135			9	+144	+261
30	4241*	+ 77*	7.950*	14.29*	4*	+ 81*	+150*
32	4540	−101			3	− 98	−205
34		+182			2	+184	+362
36	4161	+224	8.020	13.89	1.5	+226	+446
40	4075	+218	8.042	13.36	5	+223	+431
50	4352	− 64	7.901	14.96	10	− 54	−138
60	non oscillatory						
70	non oscillatory						
80	4908	−124	8.521	20.52	12	−112	−260
90	4990		8.666				
100	4937	−132	8.803	21.46	12	−120	−276
Chrom-Iron % Cr.							
10	5290	−153	7.619	21.32	10	−143	−316
20	5448	− 90	8.028	23.82	10	− 80	−190
30	5392	−102	7.659	22.27	10	− 92	−214
40	4329	− 91	7.514	21.33	10	− 81	−192

* After dipping in liquid air.

TABLE VII—*Continued.*

Material			v m/sec	g $\times 10^6$	ρ g/cc	$E \times 10^{11}$ Dyne / sq. cm.	a $\times 10^6$	h $\times 10^6$	b $\times 10^6$
Si	Cr	Fe %							
5	5	90	5166	−118	7.559	20.17			
5	10	85	5285	−111	7.581	21.17			
5	15	80	5192	−112	7.482	20.17			
5	20	75	4806	−144	7.488	17.30			
5	25	70	5473	−140	7.405	22.18			
5	30	65	5387	−133	7.414	21.52			
5.16	4.53	90		− 82					
Carbon-Steel % C									
.8			5233	−110	7.849	21.49	11	− 90	−231
1.0			5209	−137	7.827	21.24	11	−126	−285
1.5			5217	−123	7.866	21.41	11	−112	−257

In practice I no longer use directly the wavemeter or the audio-frequency meter for accurately measuring wavelengths or frequencies, but instead beat everything against the rods or piezo-crystals, and use the audiofrequency meter for measuring differences of frequencies, and the wavemeter for identifying harmonics.

Study of Frequency, Sound-Velocity, and Temperature Coefficients of Frequency of Various Alloys.—I have investigated a large number of alloys for their magnetostrictive properties, and, by measuring the frequency of oscillation of a known length of specimen, have determined the velocity of a longitudinal sound wave in these various materials. These are recorded in Column II, Table VII. By making these measurements at various temperatures I have obtained the temperature coefficient of frequency (Column III). By the formula

$$v = \sqrt{\frac{\text{Young's modulus}}{\text{Density}}}$$

I have computed Young's modulus (Column V) from measured values of velocity and density (Column IV). This method gives high precision, and is interesting in that the value of the modulus is obtained for very small applied forces. Additional data of Table VII are described below.

Other alloys are under investigation and will be reported in a later publication. Many of the specimens of the table contain a small quantity of manganese (about .3%) added to make the specimen malleable.

Curves showing Results for Velocity and Temperature Coefficients of Frequency with Alloys of Various Compositions.—Fig. 7 shows velocity of sound in *km/sec* and also temperature coefficient of frequency of vibration with rods of nickel-steel plotted against

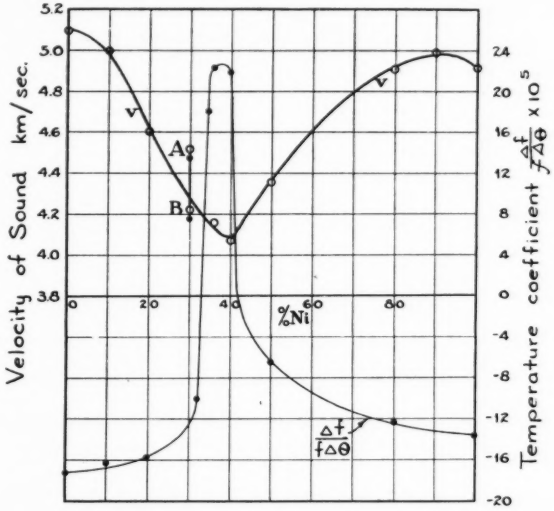


FIGURE 7. Velocity and Temperature Coefficient of Frequency of Nickel-Iron Alloys of Various Percentages of Nickel.

percentages of nickel of the rods. It is seen that the velocity has a minimum and the temperature coefficient has a maximum at 36% to 40% nickel. This is at about the composition of Invar (36% Nickel). Fig. 8 contains corresponding curves for chromium-steel of various percentages of chromium; Fig. 9, similar results for various percentages of chromium in chromium-steels which contain also 5% silicon. In Fig. 8 the velocity is a maximum and the temperature coefficient of frequency is a maximum at 20% to 25% chromium. These results, Fig. 7 and Fig. 8, suggests as a possible law *that in a*

binary metallic alloy the temperature coefficient of frequency is a maximum or minimum at the composition at which the velocity is a maximum or minimum, or the reverse. Having found a means of extending the methods of the present research to non-magnetic alloys also I have assigned to a research student the problem of testing this apparent law for a wide variety of alloys.

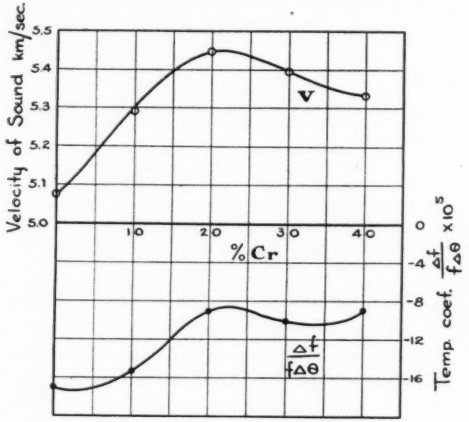


FIGURE 8. Velocity and Temperature Coefficient of Frequency of Chromium-Iron Alloys with Various Percentages of Chromium.

The curves of Fig. 9 show the effects of adding a third component, 5% silicon, to the chromium-iron series. This added silicon introduces a large minimum of velocity at 20% chromium, 5% silicon, and 75% iron. Indications of corresponding maxima or minima of the temperature coefficient of frequencies at compositions giving a maxima or minima of velocity are also apparent in these curves, but the compositions employed were not selected closely enough together to make the result certain.

An interesting result is the largeness of the change of velocity with change of composition.

The points marked *A* and *B* of Fig. 7 were obtained with a nickel-steel of 30% nickel. The values at *A* were obtained with the annealed specimen, and the interesting fact was noted that when the specimen

was heated to about 50° C. it became non-magnetic and ceased to oscillate. When it was cooled down to 20° C. it again became magnetic and oscillatory, but reversibly lost or gained its magnetism as the 50° C. point was passed. This fact for about this composition has been previously observed by Hopkinson (see Ewing: Magnetic Induction in Iron and other Metals, Van Nostrand, 1900). Acting on the facts there given regarding effects of cooling, I had my specimen dipped into liquid air. After coming back to room temperature

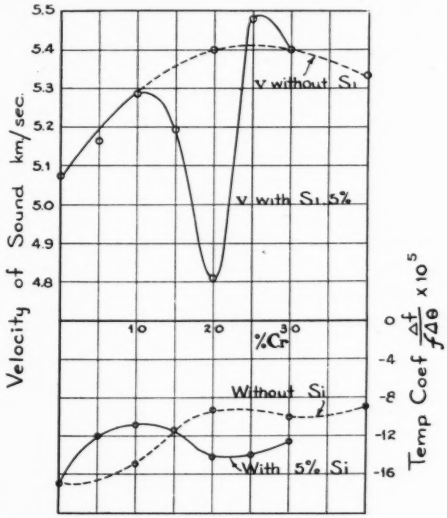


FIGURE 9. Effect of Adition of 5% Silicon to Chromium-Iron Alloys of Various Percentages of Chromium (Alscissae) and Rest Iron.

it gave the points marked *B* in Fig. 7. It had become a new substance and no longer lost its magnetism with change of temperature over the range up to 100° C.

Temperature Coefficients of Elasticity and of Velocity.—Simple relations exist among various temperature coefficients, so that other coefficients may be obtained from the experimental data of Table VII combined with known quantities taken from the work of previous investigators.

Given $\Delta\theta$ = increment of temperature,

$$a = \frac{\Delta l}{l\Delta\theta} = \text{coefficient of linear expansion,}$$

$$b = \frac{\Delta E}{E\Delta\theta} = \text{coefficient of Young's modulus } E,$$

$$g = \frac{\Delta f}{f\Delta\theta} = \text{coefficient of frequency of longitudinal vibration,}$$

$$h = \frac{\Delta v}{v\Delta\theta} = \text{coefficient of velocity of longitudinal wave,}$$

and

$$2lf = v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{EV}{W}}. \tag{1}$$

In this equation (1)

l = length of a specimen, which has
f = frequency
ρ = density,
E = Young's modulus,
W = weight,
V = volume.

If the specimen satisfying equation (1) is raised 1° C. in temperature W does not change and we obtain

$$2lf(1 + a)\,(1 + g) = v(1 + h) = \sqrt{\frac{EV}{W}(1 + b)\,(1 + 3a)}$$

Dividing this by (1) and performing the indicated operations for small values of the coefficients we obtain

$$1 + a + g = 1 + h = 1 + \frac{b}{2} + \frac{3a}{2}$$

$$\therefore \qquad h = a + g, \text{ and } b = 2g - a. \tag{2}$$

Equation (2) enables us to compute the temperature coefficient of elasticity (b) and the temperature coefficient of velocity (h) from the measured values of the temperature coefficient of frequency (g) and values of temperature coefficient of expansion (a). Since the latter are small in comparison with g they are given sufficiently accurately in ordinary tables of constants.

Values of a taken from *Bureau of Standards Circular No. 58* and from *International Critical Tables* are entered as Column VI of Table VII, and computed values of h and b are entered in Columns VII and VIII.

Figure 10 is a plot of E from my data on sound and also a plot of the corresponding values (Curve B) from the *Bureau of Standards Circular No. 58*. The large differences between these curves may be

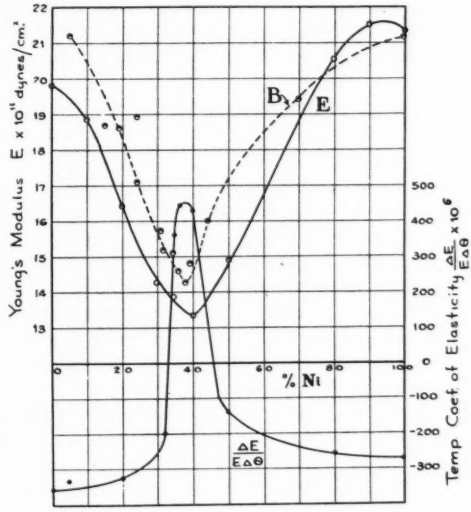


FIGURE 10. Young's Modulus and Temperature Coefficient of Elasticity of Nickel-Iron Alloys with Various Percentages of Nickel.

due to the fact that the values given in the Bureau Circular are obtained at large stresses while my values are for the very small stresses of sound waves. On the other hand, small differences in composition, as the .3% manganese in my specimens, or different heat treatment, may account for the differences. It should be further noted that my specimens were magnetized.

Methods of Obtaining Magnetostriction Vibrators of Zero or Very Small Temperature Coefficients of Frequency.—In Table VII the temperature coefficients of frequency (g) of magnetostriction

vibrators of various alloys are given in column III. In particular it is seen that the nickel-iron alloys vary from

$$g = -171 \times 10^{-6} \text{ to } g = +224 \times 10^{-6}$$

and that this coefficient is zero for alloys of 33% ni and 47% ni. Unfortunately these alloys are difficult to produce with exact properties because the curve of temperature coefficient is very steep at these points. Furthermore, these materials are only very weak magnetostriction oscillators. With sufficient care in their production and heat treatment they may prove highly serviceable as oscillators of zero temperature coefficients. A further study of alloys of this class is under way.

In combinations of three metals I have found that a vanishing temperature coefficient is usually associated with non-magnetism or non-magnetostriction.

Consequently I have investigated the alternative method of composite vibrators made of different metals so arranged that the positive coefficient of one specimen counteracts the negative coefficient of another specimen, which have the merit of simplicity of construction and can be made of materials giving a strong resultant vibration.

Two kinds of composite vibrators have been employed with success:

(*a*) A central rod of negative coefficient with a rod of positive coefficient soldered end to end at each end of the central rod. This I shall call *longitudinally composite.*

(*b*) A tube of coefficient of one sign containing a tight-fitting core rod of coefficient of the opposite sign. This I call *concentrically composite.*

A number of vibrators of both these types have been made. For standards the concentrically composite type have more power in controlling frequency and are preferred. Two such vibrators consisting of nickel tubes with stoic metal cores driven into them at a tight press-fit were found to have coefficients

$$\frac{\Delta f}{f \Delta \theta} = -\frac{1}{50000} \text{ and } -\frac{1}{65000}$$

respectively. Smaller values of this coefficient can be obtained by better choice of relative diameters of the components.

It should be noted that the cause of the change of any of these vibrators with change of temperature is the temperature coefficient of elasticity of the materials and not the change of length or change of density, both of which are relatively small.

Elementary Theory of Dynamic Magnetostriction.—As a simplification of the more exact treatment of the subject, such as is given in Appendix I, there is here presented a discussion based on the assumption that the rod acts mechanically as a pair of masses separated by an elastic member and damped by friction proportional to velocity, so as to satisfy the equation

$$f' = m\ddot{x} + r\dot{x} + \varepsilon Sx/l. \tag{1}$$

In this equation the dots indicate time derivatives, and r, m, and ε are equivalent values of frictional resistance, mass, and Young's modulus respectively,

S = Area of cross section

l = half length of the rod

x = increase of length of the half rod-length due to the force f'.

The force f' is a periodic magnetostrictive force due to a periodic magnetic induction B of comparatively small amplitude superposed on a fixed induction B_o, which is used to polarize the specimen.

Law I of Magnetostriction.—As a first law of magnetostriction we shall assume that the magnetostrictive force f' caused by the periodic induction B is proportional to B and to the area of cross section S; that is,

$$f' = aBS, \tag{2}$$

where a (positive for some substances and negative for others) is a coefficient depending on B_o but otherwise assumed constant.

We shall regard B as made up of two parts B_i and B'

$$B = B_i + B', \tag{3}$$

where B_i = induction due to the periodic current i,

B' = induction due to the reverse magnetostriction effect (Law II).

Law II of Magnetostriction.—When the rod is stretched an amount $2x$ with an original rod-length $2l$ there is produced an induction B' proportional to x/l; that is,

$$B' = a'x/l. \tag{4}$$

Solution for Time-Derivative of x.—The substitution of (2), (3), and (4) into (1) gives

$$aSB_i = m\ddot{x} + rx + \{e - aa'\}x/l, \tag{5}$$

of which the solution for the time-derivative of x on the assumption that B_i involves the time only in a factor of the form $e^{j\omega t}$ is

$$\dot{x} = \frac{aSB_i}{r + j\{m\omega - \varepsilon' S/l\omega\}} \tag{6}$$

where as an abbreviation

$$\varepsilon' = \varepsilon - aa'. \tag{7}$$

Electrical Equations.—We shall next write the electrical equations. The alternating current i is related to the impressed $e.\ m.\ f.\ e$ by the equation

$$e = Ri + \frac{d}{dt}\{\text{Flux linkage of } (B_i + B')\}. \tag{8}$$

The first two terms of the right-hand side of this equation give the electromotive force if the rod were damped so that it did not move. The remaining term gives the electromotive force due to the motion, which we shall call *motional e. m. f.* and shall designate by e', so

$$e' = \frac{d\phi}{dt}, \text{ where} \tag{9}$$

$$e' = \text{damped } e.\ m.\ f.,$$

$$\phi = \text{Flux linkage of } B'.$$

If now we write

$$\phi = a''B' \quad \text{and} \tag{10}$$

$$B_i = L'i, \tag{11}$$

where a'' and L' are constants depending on the size, shape, and number of turns of the coil and on the permeability of the specimen, and combine the equations (4), (6), (9), (10), and (11), we obtain

$$e' = z'i, \text{ where} \tag{12}$$

$$z' = \frac{aa'a''L'S/l}{r + j\{m\omega - \varepsilon' S/l\omega\}}. \tag{13}$$

In equation (13) z' is the complex motional impedance of the system. ω is the angular velocity of the current. a, a', a'' and L' are coefficients defined by (2), (4), (10), and (11) respectively.

This theory and the discussion of the next section is essentially of the same character as that given by Professor Kennelly and me for the impedance of telephone receivers (*Proc. Am. Acad.*, Vol. 48, No. 6, 1912, and *Electrical World*, September, 1912.)

Impedance Circle Diagrams.—If we let

$$R' = \text{motional resistance},$$

$$X' = \text{motional reactance}$$

of the rod-coil unit, and as further abbreviations, write

$$A = aa'a''L'S/l \quad \text{and} \tag{14}$$

$$Z_m^2 = r^2 + \{m\omega - \varepsilon' S/l\omega\}^2, \tag{15}$$

we find that equation (13) after rationalization gives

$$R' = \frac{Ar}{Z_m^2} \tag{16}$$

$$X' = \frac{-A\{m\omega - \varepsilon' S/l\omega\}}{Z_m^2} \tag{17}$$

whence

$$R'^2 + X'^2 = A^2/Z_m^2.$$

The substitution of (16) so as to eliminate Z_m gives

$$R'^2 + X'^2 = AR'/r. \tag{18}$$

This is the equation to a circle, passing through $R' = 0$ and $X' = 0$, and having a diameter A/r, and is represented as the left-hand circle of Fig. 11. Each value of ω gives a point of the circle. The circle is thus the locus of X' plotted against R', or, otherwise stated, it is a vectorial plot of z' as ω varies.

Impedance Circle-Diagrams With Diameter Dipping.—In the preceding section the diameter of the Impedance Circle lies along the axis of motional resistance. Experimentally this is found to be not the fact. The diameter of the circle usually dips by an angle which we shall call β.

This result can be obtained theoretically if we let one or more of the quantities a, a', a'', L' be a complex quantity independent of ω so that

$$Ae^{-j\beta} = aa'a''L'S/l. \tag{17}$$

This is equivalent to multiplying every vector z' by $e^{-j\beta}$ which gives the whole diagram the form of the right-hand circle of Fig. 11.

As to the physical significance of complex values of a, a', a'', or L', which may be inferred from the experimentally determined values of impedance circles, it seems probable that a and a' are the complex quantities and that their complexity is due largely to hysteresis effects and eddy-currents in the magnetostrictive core as the dip angle largely disappears when the core is a thin nickel tube split longitudinally.

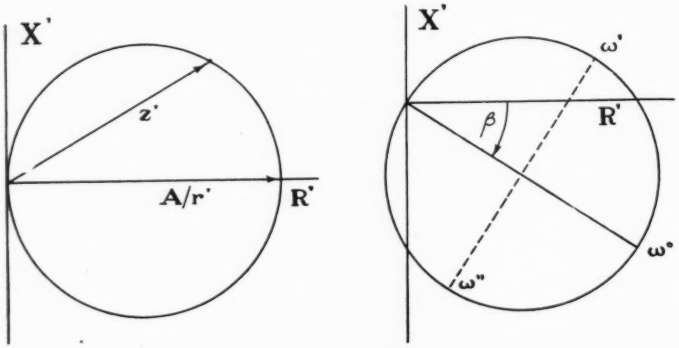


FIGURE 11. Impedance Circle Diagrams.

An extensive experimental study of motional impedance in the case of magnetostriction has been made at this laboratory by Mr. K. C. Black at my suggestion and is contained in his thesis for the Doctorate of Philosophy deposited in manuscript in the Harvard Library. He is publishing a short account of this investigation in a paper following this in the Proceedings of the American Academy.

The results are very remarkable in showing that the magnetostrictive system has a very high efficiency as a converter of electric energy into mechanical motion.

Magnetostrictive Sources of Sound.—I have made a number of applications of magnetostriction to the production of sound and

ultra sound over a wide range of frequencies. These devices are outside of the scope of the present paper. One application, which is to the measurement of the velocity of sound at various frequencies, is being employed by some of my graduate students in filling in the gaps of the work I previously published on Proc. Am. Acad. Vol. 60, No. 5, 1925, in which a piezo-electric crystal oscillator was the radiator of the sound, and the reaction of the reflected sound on the source was the means for indicating nodes and loops of the standing wave system. The frequencies used in those experiments extended from 40,000 to 1,400,000 cycles per second. The use of the magnetostriction vibrators in the same general arrangement of apparatus permits the use of frequencies below the 40,000 cycle value, and also, since rods are easily cut to any predetermined frequency, permits filling in certain significant frequencies that were lacking in the previous investigations. Also various gases are being employed in the experiments.

Another sound-velocity application consists in putting various solid non-magnetostrictive rods on the end of a magnetostrictive driving rod, and by means of frequency measurements determining sound-velocity and elastic properties of non-magnetic and other non-magnetostrictive materials. These new investigations are well under way.

Arrangements for Obtaining High Frequencies.—In the production high-frequency stabilization by magnetostriction rods I have used three methods. (*a*) One is by the use of very short cylinders, placed within the plate coil with their axes parallel to the magnetic field of this coil; (*b*) another is by flat strips or sheets of the magnetostrictive material placed between the plate and grid coils; and (*c*) by what I may call *beaded rods*. The latter method is probably the best. An illustrative example, shown in Fig. 12, consists of a rod 1.26 cm. in diameter and 9 cm. long with grooves turned in it to a depth of half the radius. The grooves are 1 cm. wide and separated by one centimeter, so that the vibrator is in the form of five cylindrical beads, each one centimeter long held in succession one centimeter apart by cylindrical rods of half the diameter of the beads.

When this beaded rod has been annealed and is placed in a coil-pair having 200 turns on each coil, and so positioned that one bead is in one coil and the next bead is in another coil or else one bead between coils, it shows strong stabilization of frequency 295,480 cycles per second. The material is stainless steel with velocity of 5430 meters per second, so the frequency is that of a linear vibrator

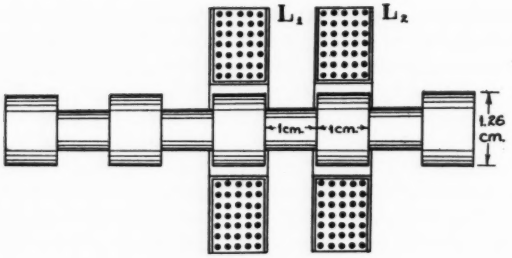


FIGURE 12. Beaded Rods.

of length $543000 \div 2 \times 295480 = .918$ cm., which is about 10% less than the computed value of one of the beads standing alone and regarded as a linear vibrator. There are other frequencies of the system particularly one at 19888 cycles per cecond.

The theory of such a beaded vibrator has been pretty well worked out but to save space is not here given.

For lack of space also the general vacuum-tube theory is here omitted.

In the accompanying appendix a more detailed theory of sound propagation in the magnetostriction rod is presented.

APPENDIX I.

Theory of Dynamic Magnetostriction, Including Theory of Propagation of Sound in a Viscous Magnetostrictive Medium.— Referring to Fig. 13, let us consider a rod of length $2l$ wound with a solenoid of N turns per centimeter.

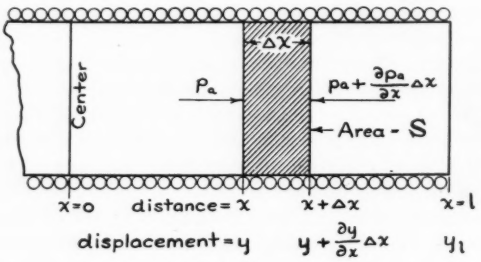


FIGURE 13.

Let S = area of cross section of the rod. Let x = the distance from the center to any section of the rod, and let us consider a volume of length Δx extending from x to $x + \Delta x$.

Let p_a and y = external pressure and displacement at x of the section Δx, and

$$p_a + \frac{\partial p_a}{\partial x}\Delta x \text{ and } y + \frac{\partial y}{\partial x}\Delta x = \text{the corresponding quantities at } x + \Delta x,$$

and let us note that the pressure at x and $x + \Delta x$ are both estimated inward upon the section Δx, while the displacements are both in the x-direction.

Before analysing the external pressure p_a into its components, we may equate the force in the x-direction to the mass $\times$ acceleration of the elemental volume Δx, obtaining

$$(1) \qquad S\left[p_a - \left\{p_a + \frac{\partial p_a}{\partial x}\Delta x\right\}\right] = S\Delta x \rho \frac{\partial^2 y}{\partial t^2},$$

in which ρ = density of the material of the volume element.

Performing the operations of (1), dividing by $S\Delta x$ and taking the limit as Δx approaches zero, we obtain

$$(2) \qquad -\frac{\partial p_a}{\partial x} = \rho \frac{\partial^2 y}{\partial t^2}.$$

Let us now analyse p_a into its constituents; namely, let

$$(3) \qquad p_a = p_\varepsilon + p_G + p_m, \text{ where}$$

p_ε = pressure due to elasticity,

p_G = pressure due to viscosity,

p_m = pressure due to magnetostriction.

These various p's are due to the pressures of expansion of an element to the left of that shown in the figure, and are given by the equations

$$(4) \qquad p_\varepsilon = -\varepsilon\left\{\frac{\text{increase of volume}}{\text{volume}}\right\} = -\varepsilon\frac{\partial y}{\partial x},$$

$$(5) \qquad p_G = -G\frac{\partial}{\partial t}\left\{\frac{\text{increase of volume}}{\text{volume}}\right\} = -G\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial x}\right),$$

$$(6) \qquad p_m = a\,B, \quad \text{in which}$$

ε = Young's modulus in dynes per sq. cm.

G = a viscosity coefficient in *c. g. s.* units,

B = magnetic induction at x, producing p_m,

a = a coefficient relating expansive pressure of magnetostriction in dynes per sq. cm. to magnetic induction B.

The coefficients ε and G are positive, a is positive for some magnetostrictive materials and negative for others. Also a is not exactly constant but varies with B as well as with the material. We shall, however, need to consider it a constant where B is a small induction added to an already existent constant large induction.

The substitution of (4), (5), and (6) into (2) and (3) gives

$$\rho \frac{\partial^2 y}{\partial t^2} = \varepsilon \frac{\partial^2 y}{\partial x^2} + G \frac{\partial}{\partial t} \frac{\partial^2 y}{\partial x^2} - a \frac{\partial B}{\partial x}, \tag{7}$$

and

$$p_m = - \varepsilon \frac{\partial y}{\partial x} - G \frac{\partial}{\partial t} \frac{\partial y}{\partial x} + aB. \tag{8}$$

We shall now further analyse B. We shall suppose the existence of a steady magnetic induction B_o, which will polarize the system and determine the size of the coefficient a, but will not contribute to p_m, which we shall suppose to be due to an additional periodic B. This additional B is made up of a quantity due to the variable current i in the windings plus an additional quantity B' called into play by magnetostriction as a result of distortion; that is

$$B = B_i + B' \quad \text{where} \tag{9}$$

B_i = magnetic induction due to i,

B' = magnetic induction due to expansion of the element Δx and is a function of x.

This B' we shall assume proportional to increase of length of the element divided by the original length of the element; that is,

$$B' = a' \frac{\partial y}{\partial x}. \tag{10}$$

In substituting (9) and (10) into (7) and (8) we shall regard

$$\frac{\partial B_i}{\partial x} = 0. \tag{11}$$

The substitution then gives

$$\rho\frac{\partial^2 y}{\partial t^2} = \varepsilon'\frac{\partial^2 y}{\partial x^2} + G\frac{\partial}{\partial t\partial x^2} \tag{12}$$

and

$$p_a = -\varepsilon'\frac{\partial y}{\partial x} - G\frac{\partial}{\partial t\partial x}\frac{\partial y}{\partial x} + aB_i, \tag{13}$$

where as an abbreviation

$$\varepsilon' = \varepsilon - aa'. \tag{14}$$

Equations (12) and (13) are to be treated as simultaneous.

In solving (12) we shall here restrict the treatment to the case in which the time t enters in y only in a factor of the form $e^{j\omega t}$, where ω is the angular velocity of the current i. This reduces (12) to the form

$$-\rho\omega^2 = (\varepsilon' + jG\omega)\frac{\partial^2 y}{\partial x^2}, \tag{15}$$

which gives

$$y = e^{j\omega t}\{Ae^{kx} + A'e^{-kx}\} \tag{16}$$

with

$$k = +j\omega\sqrt{\frac{\rho}{\varepsilon' + jG\omega}}. \tag{17}$$

We shall next stipulate that the center of the rod has zero motion; that is, $y = 0$ at $x = 0$. This gives

$$y = A\, e^{j\omega t}\sinh kx. \tag{18}$$

Now to determine A, we introduce the further boundary condition that $p_a = 0$ at $x = l$, so (13) gives

$$0 = -(\varepsilon' + jG\omega)\frac{\partial y}{\partial x} + aB' \quad \text{at } x = l. \tag{19}$$

Replacing y in this equation by its value from (18) we obtain

$$Ae^{j\omega t} = \frac{aB'}{k(\varepsilon' + jG\omega)\cosh kl},$$

and consequently by (18)

$$y = P\sinh kx, \tag{20}$$

where as an abbreviation

$$P = \frac{aB'}{k(\varepsilon' + jG\omega)\cosh kl}. \tag{21}$$

Let us now turn to a consideration of the electrical system in which there is a variable current i flowing. The variable *e. m. f.* at the terminals of the coil is

$$e = Ri + \frac{\partial}{\partial t}\{\text{flux linkage of } (B_i + B')\}. \tag{22}$$

The first two terms of the right-hand side of this equation give e damped; that is, the value of e if the rod were restrained from vibrating. The last term is the contribution of the motion, and may be called the *motional e. m. f.*, so

$$e' = \frac{\partial \varphi}{\partial t}, \quad \text{where} \tag{23}$$

e' = motional *e. m. f.*,

φ = flux linkage of B' with the circuit.

To obtain φ, let us note that per unit area of S at any element Δx, distant x from the center, magnetic induction of amount $B' - (B' + \Delta x \partial B'/\partial x)$ leaks out between the turns of wire and these return to the core at distance minus x from the center, and thus link with $2Nx$ turns giving

$$\Delta\varphi = -2NSx\frac{\partial B'}{\partial x}\Delta x,$$

where N = number of turns of wire per centimeter.

To get the total linkage we must integrate this from $x = 0$ to $x = l$, and must add $2NSlB_l'$ to account for leakage from the end, obtaining

$$\varphi = 2NS\left\{-\int_0^l x\frac{\partial B'}{\partial x}dx + lB_l'\right\}. \tag{24}$$

We may first integrate this by parts by putting

$$u = x,\ dv = \frac{\partial B'}{\partial x}dx, \text{ obtaining}$$

$$\varphi = 2NSl\int_0^l B'\,dx.$$

We may now complete the integral by replacing B' by its value from (10) and obtain

(25) $\varphi = 2a' NS\, y_l$, where

y_l = value of y at l.

By (20) and (21) this gives

$$\varphi = 2a' NSP \sinh kl$$

$$= \frac{2aa' NSB_i}{k(\varepsilon' + jG\omega)} \tanh kl.$$

Now returning to the motional *e. m f.* equation (23), and noting that the time derivative is $j\omega$ times the quantity, and then replacing $j\omega/k$ by its value from (17), we obtain

(26) $$e' = \frac{2aa' NSB_i}{\sqrt{\rho\{\varepsilon' + Gj\omega\}}} \tanh kl.$$

If now

L = self inductance of the coil with rod damped, we may write Li = flux linkage of Bi, which is $2NSlB_i$, so

(27) $$B_i = \frac{Li}{2NSl},$$

which substituted into (26) gives

(28) $$e' = Qi \tanh kl.$$

In this equation I have introduced as an abbreviation

(29) $$Q = \frac{aa' L}{l\sqrt{\rho\{\varepsilon' + jG\omega\}}}.$$

Motional Impedance.—If we let z' = complex motional impedance of the system, and compare (28) we have

(30) $$z' = Q \tanh kl.$$

Equation (30) gives the complex motional impedance of the system, Q and k are defined by (29) and (17) respectively.

To investigate the manner in which z' varies with ω, we may note that while Q involves ω, the term in which the ω appears is comparatively small, and since the whole motional action is confined to a very small range of ω, we may regard Q as a constant.

Let us now write

$$k = \alpha + j\beta, \text{ where} \tag{31}$$

$$\alpha = \omega \sqrt{\frac{\rho}{\varepsilon'(1 + h^2)}}\, g(h), \tag{32}$$

$$\beta = \omega \sqrt{\frac{\rho}{\varepsilon'(1 + h^2)}}\, f(h), \tag{33}$$

$$f(h) = \sqrt{\frac{\sqrt{1 + h^2} + 1}{2}}, \tag{34}$$

$$g(h) = \sqrt{\frac{\sqrt{1 + h^2} - 1}{2}}, \tag{35}$$

$$h = G\omega/\varepsilon'. \tag{36}$$

Regarding the functions $f(h)$ and $g(h)$ see G. W. Pierce: *A Table and Method of Computing Electric Wave Propagation, Transmission Line Phenomena, Optical Refraction, and Inverse Hyperbolic Functions of a Complex Variable, Proc. Am. Acad., Vol. 57. No. 7, 1922*, where a table of the functions f and g is given for a large range of values of h.

Introducing the value (31) for k into (39) and breaking z' into its resistance and reactance components, R' and X', we have

$$z' = R' + jX' = Q \frac{\sinh (\alpha + j\beta)\, l}{\cosh (\alpha + j\beta)\, l}$$

which after expansion and rationalization becomes

$$R' + jX' = Q \frac{\sinh \alpha l \cosh \alpha l + j \sin \beta l \cos \beta l}{\sinh^2 \alpha l + \cos^2 \beta l}.$$

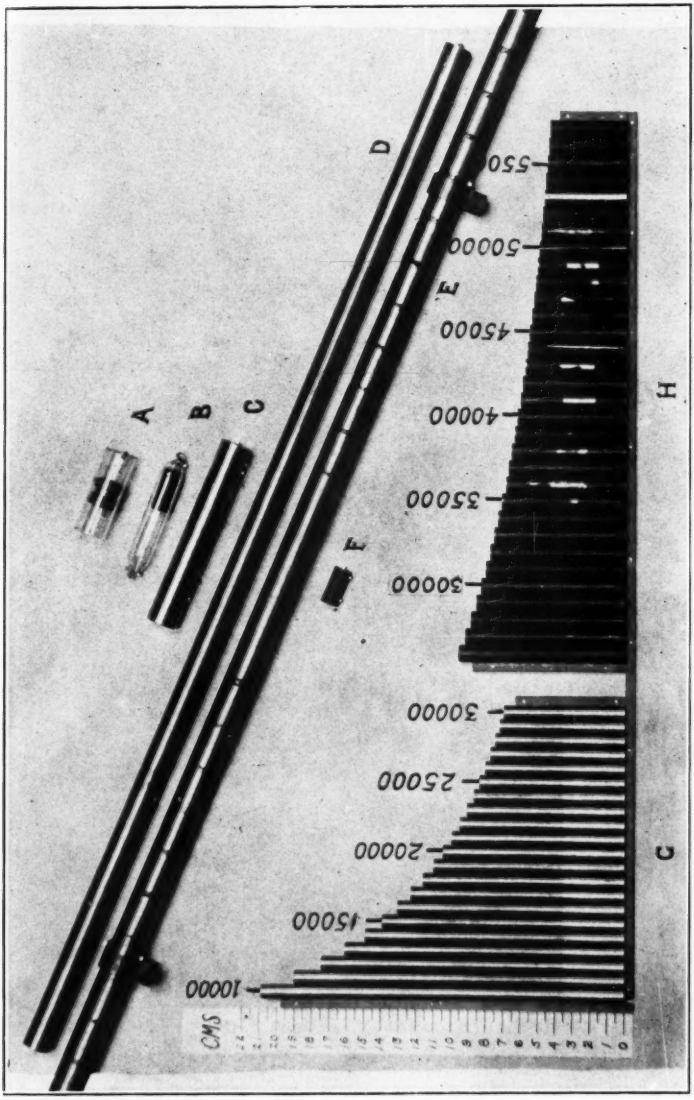
From this we may obtain

$$R'^2 + X'^2 = 2R_0 R', \qquad \text{where} \tag{37}$$

$$2R_0 = Q \frac{\sinh^2 \alpha l + \sin^2 \beta l}{\sinh \alpha l \cosh \alpha l}. \tag{38}$$

Equation (37) is in the form of the equation of a circle of radius R_o. Equation (38) shows, however, that R_o involves the variable parameter ω so (37) is not a true circle. Nevertheless, in the case of the magnetostrictive system, the resonance is so sharp that the whole phenomenon of motional reactance is exhibited within a frequency range of a fraction of one percent of the resonant frequency; so the variation of the coefficient R_o is inappreciable to the limit of the errors of the experimental measurements that Dr. Black has made and is publishing in a succeeding number of these proceedings.

CRUFT LABORATORY,
Harvard University, Cambridge, Mass. U.S.A.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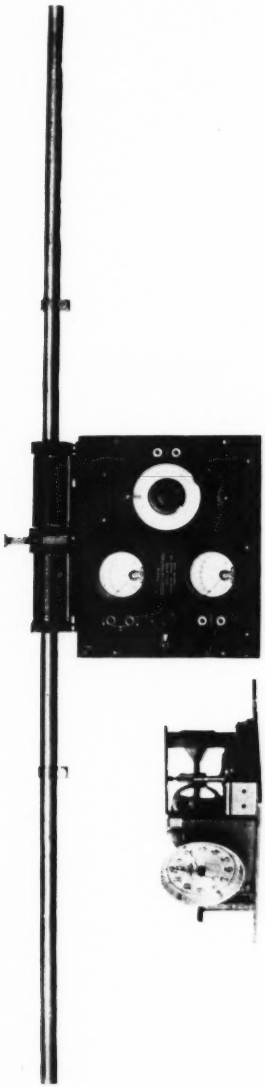

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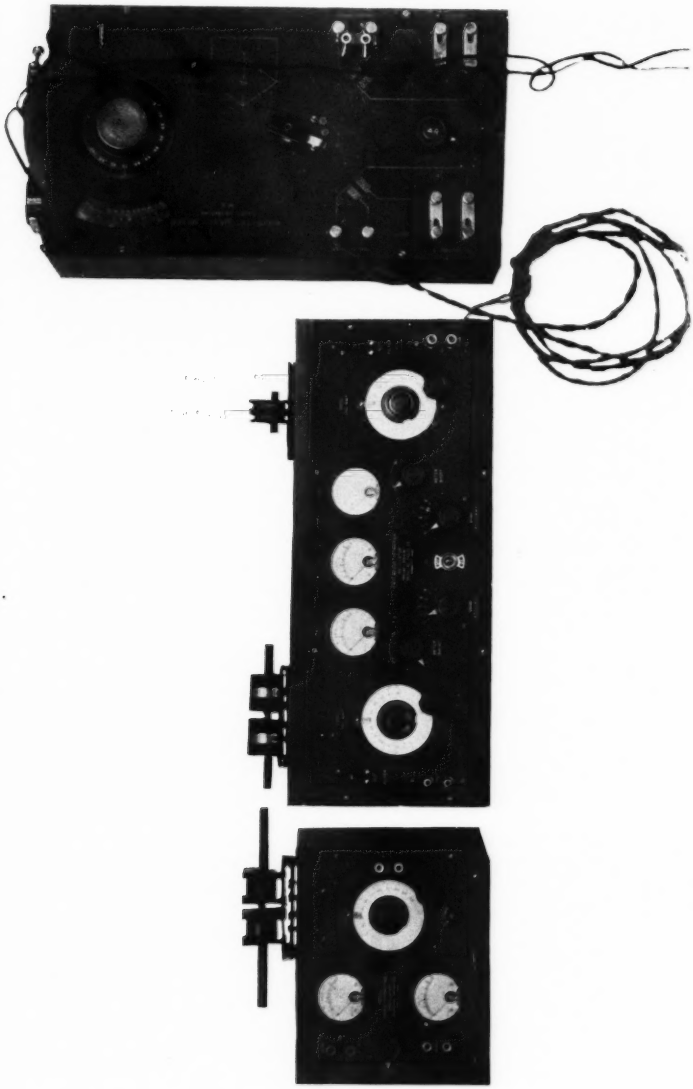

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